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HIGHER ORDER NUMERICAL HOMOGENIZATION IN MODELING OF ASPHALT CONCRETE[1]

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In this paper, we present an enhanced version of the two-scale numerical homogenization with application to asphalt concrete modeling in the elastic range. We modified the method of effective material parameters tensor assessment for analysis based on the representative volume element (RVE). As the method was tested on asphalt concrete, we also present two possible approaches to geometrical modeling of its internal microstructure. Selected numerical tests were performed to verify the proposed approach. The main novelties of this study, i.e. higher order approximation at the macroscale and modification of boundary conditions at the level of RVE analysis, improved the efficiency of our methodology by error reduction. Practically, we obtained a reduction of NDOF up to 3 orders of magnitude (comparing to full-scale and homogenized analysis) that was accompanied with the introduced error of order of several percent (measured in L_2 norm).

Keywords: asphalt concrete, numerical homogenization, representative volume element

1. Introduction

Roads are systematically classified as "linear infrastructure objects". Practically, they exhibit a fully three-dimensional structure as multi-layered domains. In Poland, the roads with flexible pavement structures still remain the most popular among other types of pavements, i.e., rigid or semi-rigid ones. A flexible pavement structure consists of several asphalt layers and subbase layer(s) made of crushed rock resting on an improved subgrade. Due to specific groundwater or ground conditions, some additional layers (e.g., those made of geosynthetics) may be also applied.

The upper layers of the asphalt pavement structure play different roles in providing the demanded bearing capacity, durability and other parameters of the whole structure. Consequently, their internal structures are also different. It is mainly due to variety of asphalt mixture types that can be selected: asphalt concrete (AC), stone-mastic-asphalt (SMA), hot-rolled asphalt (HRA), reclaimed asphalt pavement (RAP), to mention only a few. The diversity within a specific mixture type can be obtained using extra additives or modifying the gradation curve of the aggregate mixture.

In this paper, we focus on the selected aspects of asphalt concrete modeling. In particular, the asphalt concrete microstructure and its impact on the effective macroscale parameters is studied. In Fig. 1, one can observe the limiting gradation curves of this asphalt mixture used for different pavement layers (GDDKiA, 2014). It can be noticed that the gradation is coarser for the bottom layers than for the upper ones. It is not only exclusively due to the fact that thickness of the subbase is greater than that of the wearing and binder course. Finer aggregate is used for the wearing course to resist visco-plastic deformations (ruts) occurring herein. Consequently, a coarser aggregate is used in the subbase in order to reduce the risk of structural deformations.

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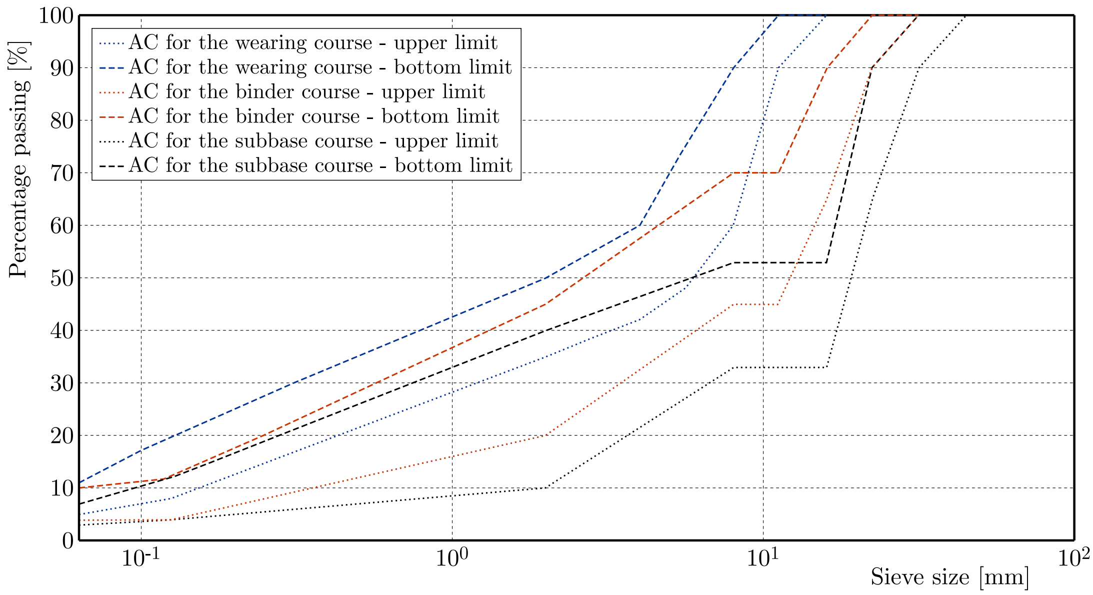

Fig. 1. AC gradation curves for different pavement layers plotted in a semilogarithmic scale (traffic category KR3-KR6)

As a material, asphalt concrete is a mixture of two main phases: aggregate particles and a mastic (which is, in turn, a mixture of the bitumen binder and mineral filler). The weight ratio of these two main phases is greater than 90/10. Volumetrically, several percent of air voids can be also distinguished. The description presented above is a rough approximation of the asphalt concrete recipe in its traditional form. There is a very active research field devoted to neat asphalt modifications, mastic modifications as well as the replacement of the natural aggregate mixture with different industrial wastes in the spirit of less-waste philosophy (Fakhari Tehrani *et al.*, 2013; Kim *et al.*, 2013; Schüller *et al.*, 2016; Ziaei-Rad *et al.*, 2012).

Aiming at the reliable numerical analysis of asphalt concrete, one needs to decide on the number of aspects. Let us list a three of them that are fundamental in our opinion:

- *Analysis scale* – from atomistic to the specimen/pavement structure scale (called in this paper as the macroscale). It is noticeable that the macroscale response is highly related to phenomena observed at the lower scales. In this paper, we present the framework for multiscale analysis that bridges the macroscale with the asphalt mixture scale (referred to as the microscale). We decided to keep this nomenclature for the sake of brevity. In the literature, the asphalt mixture scale is sometimes referred to as the mesoscale, whereas the microscale term is reserved for the scale of mastic with particles of dimensions of several μm. In this distinction, however, it would be difficult to describe the scale of asphalt mortar (with particles smaller than 2 mm). The sequence of consecutive analysis scales is shown in Fig. 2. Summing up, we model the specimen at the macroscale with its spatial dimensions kept but with the assumption on the homogeneity of the domain. Its effective parameters are numerically assessed on the basis of the microscale analysis. Precisely, we use the specific material parameters for the inclusions and for the matrix at the microscale and compute effective macroscale quantities.
- *Material model* – for every analysis scale addressed in this paper, one can easily identify the heterogeneity of its underlying scale. Regardless of the selected material model at the specific scale, the material response is evidently affected by lower scale phenomena. In the case of asphalt concrete, the constitutive equations describing the bitumen behavior are of particular interest. Linear (Aigner *et al.*, 2009; Collop *et al.*, 2003; Fakhari Tehrani *et al.*, 2013; Klimczak and Cecot, 2020a; Mo *et al.*, 2008; Woldekidan *et al.*, 2013; Ziaei-Rad *et al.*,

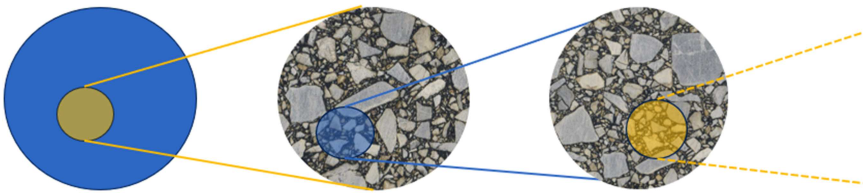

Fig. 2. Sequence of the selected analysis scales addressed in this study (yellow dotted line illustrates the occurrence of further underlying scales)

2012) and nonlinear viscoelasticity (Mitra *et al.*, 2012; Schüller *et al.*, 2016; Woldekidan *et al.*, 2013), viscoelastoplasticity (Aigner *et al.*, 2009; Collop *et al.*, 2003), viscoelasticity with damage (Kim *et al.*, 2013) and other theories are developed, to mention only a few. For the sake of brevity, we present in this study our results with the assumption on the elastic behavior of the binder phase. From the engineering point of view, this is a very strong assumption and can be understood as steady-state analysis at a specific temperature. The main scope of this paper is the application of the developed numerical method, however. Consequently, we apply both for the binder and aggregate phases the assumption on the linear elasticity in order to present the main findings of our research. Nonlinear analyses of materials can be found in our previous papers (Klimczak and Cecot, 2020b; Oleksy and Cecot, 2015).

- *Direct/multiscale analysis* – numerical analysis of the composite can be performed at a specific scale in a manifold manner. An assumption on the domain homogeneity can be made and the effective parameters are assessed phenomenologically/analytically or numerically. Consequently, the complexity of numerical analysis at this scale is very low, but the material response can be very smoothened. On the contrary, a very resource-consuming direct analysis can be performed with accounting for the internal structure of the composite (Fakhari Tehrani *et al.*, 2013; Mitra *et al.*, 2012; Mo *et al.*, 2008; Ziaei-Rad *et al.*, 2012). This type of analysis provides a deep insight into the lower scale phenomena, but it is time consuming and produces also an amount of data that can be hard to analyze and process. Somewhere in-between, there is a large group of multiscale methods. Generally, the macroscale analysis is performed at a low cost, but the microscale oscillations are accounted for performing some additional local analyses. The methods based on the concept of the representative volume element (RVE) are of particular significance nowadays.

In the field of asphalt concrete numerical modeling, one can distinguish a number of approaches to the above-mentioned aspects. A specific methodology is typically a trade-off between the complexity of the analyzed phenomena and the numerical cost of analysis. Below, we discuss several selected methodologies used in numerical modeling of asphalt concrete (or similar asphalt mixtures).

In (Collop *et al.*, 2003), the authors developed an elasto-visco-plastic model with damage for asphalt pavement layers. They modeled a multi-layered domains with the assumption on the homogeneity of every layer. The numerical effort of the analysis consisted in a time-stepping algorithm, whereas the geometry of the pavement structure was simple due to the assumed effective parameters for the whole layer. No multiscale analysis was necessary. In (Woldekidan *et al.*, 2013) and their other papers, the authors extended the scope of the analysis presented in (Collop *et al.*, 2003) and assessed the effective parameters of the asphalt concrete at different observation scales and for different material models. Such studies can be understood as a phenomenological assessment of the effective material parameters.

In (Mo *et al.*, 2008), the authors analyzed performance of porous asphalt concrete with a particular focus on the ravelling process (loss of wearing course aggregate particles). The internal structure of the domain was idealized significantly to facilitate the analysis. The authors used cylindrical (2D) or spherical (3D) objects to geometrically model the aggregate particles. They studied the interfacial zone between them and the bitumen binder using the viscoelastic material model. Since the analyzed specimens were idealized, it was possible to carry out a direct analysis. Nevertheless, it was an attempt to include in the numerical analysis the microscale phenomena. The numerical cost was reduced by geometry simplification.

Another approach to reliable modeling of asphalt concrete with a direct microstructure can be found e.g. in (Klimczak and Cecot, 2020a; Ziaei-Rad *et al.*, 2012). Therein, the authors analyze the specimens with the microstructures equivalent to the actual one. Namely, they generate synthetic microstructures possibly similar to the realistic ones. However, it is somehow arbitrary how to verify the similarity of these two types of microstructures beyond the visual inspection. The methods based on the Voronoi tessellation combined with the control of prescribed gradation curves are a typical approach. The shapes of inclusions are close to the realistic ones yet simplified. Consequently, direct transient analysis using e.g., viscoelasticity principles can be carried out at the microscale. Simplification of the aggregate particle shapes allows for a substantial reduction of the number of elements/nodes necessary for the numerical analysis. Typically, the asphalt mixture scale is used. If the contact phenomenon was also the analyzed problem, the aforementioned internal structural simplifications should refer to the respective scale. Oversimplified geometries would not cover the binder-aggregate interaction properly.

A very important and active research field is a multiscale analysis of composites. There is a wide spectrum of methods within this approach. For their comprehensive classification and review, we refer the reader e.g. to (Belytschko and de Borst, 2010; Fish, 2014; Kouznetsova *et al.*, 2002). Such a summary is beyond the scope of this paper. Instead, the applications of selected multiscale methods to the modeling of asphalt concrete and similar asphalt mixtures are briefly summarized below. The common feature of all these methods is the fact that they bridge the neighboring scales. The information derived from the lower analysis scale is transferred to the upper scale in order to facilitate computations at this level. Schematically, additional computations are necessary to incorporate the lower scale information at the upper scale. This cost, however, is justified in a manifold manner. Firstly, it is a way of making the analysis at some lower scales feasible. Secondly, the speed-up (very often due to possible parallel computing) is observed. A time-consuming direct analysis can be avoided. Thirdly, a number of neighboring scales can be analyzed simultaneously giving a deeper insight into the impact of the modifications introduced at the lower scale on the overall material response.

In (Aigner *et al.*, 2009), the authors used the concept of the localization tensor to study viscoelastic properties of asphalt concrete. Assuming spherical inclusions (aggregate particles) and using the Mori-Tanaka scheme (Mori and Tanaka, 1973) they obtained closed formulas for the effective material parameters at the macroscale. The idea of the representative volume element (RVE) was used to compute them locally. The RVE size should correspond with the size of inclusions to represent full information on the microstructure. Since asphalt concrete exhibits a random microstructure, this concept is more suitable than the unit-cell approach used for periodic domains.

In (Feyel and Chaboche, 2000; Guedes and Kikuchi, 1990; Kouznetsova *et al.*, 2002), the computational homogenization (typically associated with the finite element method) was developed and tested on various materials. Its standard version is also based on the RVE concept. In numerical analysis, two neighboring scales are specified. At the macroscale, one generates a coarse mesh and specifies a set of characteristic points (usually Gauss integration points) at this level. With each of such points, an RVE representing the local microstructure is defined. An iterative analysis is performed to transfer interchangeably the information from both scales.

First, the macroscale problem is solved. Deformation at Gauss points is used as a boundary condition for auxiliary boundary value problems solved within the RVE's corresponding with these points. Subsequently, the averaged quantities – see (Feyel and Chaboche, 2000; Guedes and Kikuchi, 1990; Kouznetsova *et al.*, 2002) for details – from this level are transferred to the macroscale and used for the next iterative solution. It should be underlined that no assumption on the material model at the macroscale is necessary in this approach, since the averaged strains and stresses are computed at the microscale level. In terms of the asphalt concrete modeling, the computational homogenization was used e.g., in (Kim *et al.*, 2013; Schüller *et al.*, 2016).

In (Wimmer *et al.*, 2016), a method of synthetic microstructure generation based on the Voronoi tessellation was presented to model the RVE. These local microstructures were later used for a randomly located set of RVE's. For each of them, periodic boundary conditions were used. Finally, the effective material parameter tensors were assessed using the statistics. The linear elastic model was used both for the matrix and the inclusions.

In (Schüller *et al.*, 2016), the effective macroscale stress using the generalized Maxwell-Zener viscoelastic model was computed on the basis of the RVE analysis. The authors also generated the Voronoi diagram-based microstructures replacing those obtained using the X-ray computed tomography (XRCT) as leading to the overkill mesh generation.

In (Kim *et al.*, 2013), the computational homogenization was used in order to bridge the effect of the cohesive zone occurrence at the microscale with damage observed at the macroscale. The macroscale stresses and strains were computed in terms of the microscale ones. Two numerical tests performed for idealized and real-like RVE microstructures illustrated the proposed framework.

Another approach to modeling of asphalt concrete was presented in (Klimczak and Cecot, 2020b). Therein, special shape functions accounting for the complex microstructure of the analyzed material were used for the solution approximation at the macroscale. This approach was successfully used for both the linear elastic and viscoelastic material models.

2. Methodology

2.1. Numerical homogenization

In this paper, we present the framework for higher order numerical homogenization of asphalt concrete. The numerical homogenization was developed e.g., in (Zohdi and Wriggers, 2005). Its idea is also based on the RVE concept. Namely, one performs a set of numerical tests for the identified RVE (Fig. 3).

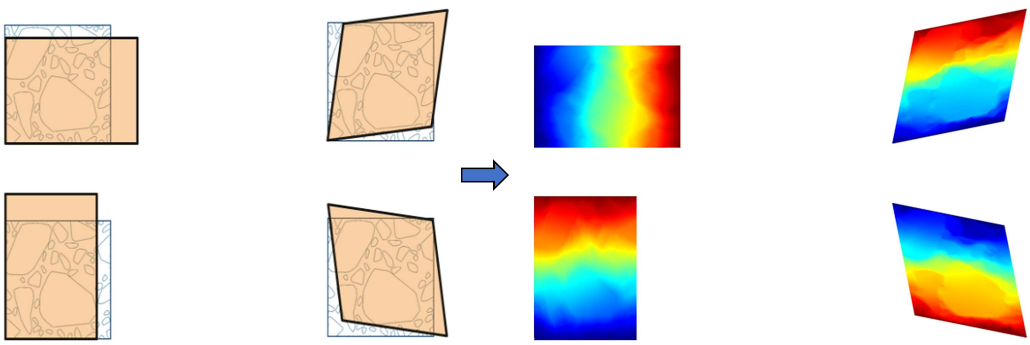

Fig. 3. A set of numerical tests (left) and their solutions (right)

Accounting for the microstructure observed at the RVE level, one solves a set of boundary value problems corresponding to real laboratory experiments (Fig. 3). Such an approach can be understood as substitution of a similar procedure realized in a laboratory (phenomenological approach). In the numerical homogenization, average strain $\langle\varepsilon_{ij}\rangle$ and stress $\langle\sigma_{ij}\rangle$ components are computed as follows for these tests

$$\langle\varepsilon_{ij}\rangle = \frac{1}{V}\int_{\Omega} \varepsilon_{ij}\, d\Omega \qquad \langle\sigma_{ij}\rangle = \frac{1}{V}\int_{\Omega} \sigma_{ij}\, d\Omega \tag{2.1}$$

Assuming the constitutive law of the form

$$\langle\boldsymbol{\sigma}\rangle = \mathbf{C}_{\mathit{eff}}\langle\boldsymbol{\varepsilon}\rangle \tag{2.2}$$

one can obtain the effective tensor of material parameters $\mathbf{C}_{\mathit{eff}}$. Depending on the expected material behavior, various forms of this tensor can be adopted. In the case of asphalt concrete, the choice between isotropy or anisotropy seems reasonable. Such effective tensors of material parameters should be assessed for every RVE. The boundary value problem at the macroscale is finally solved using these effective tensors.

2.2. Analysis enhancements

In our study, we propose additional enhancements of the methodology presented in the previous Section. Firstly, we use the higher order approximation at the macroscale level (hierarchical shape functions used). Secondly, we modify the boundary conditions used for numerical tests performed for the specified RVE. This modification is based on the observation that the numerical tests are to reflect the behavior of the heterogeneous subdomain from the interior of the whole analyzed domain. Hence, the boundary conditions should account for the heterogeneity of the material along the boundaries. The idea is schematically presented on the example of the shear test in Fig. 4. Instead of the standard boundary conditions marked with red dashed lines, we use their modified versions marked with blue continuous lines.

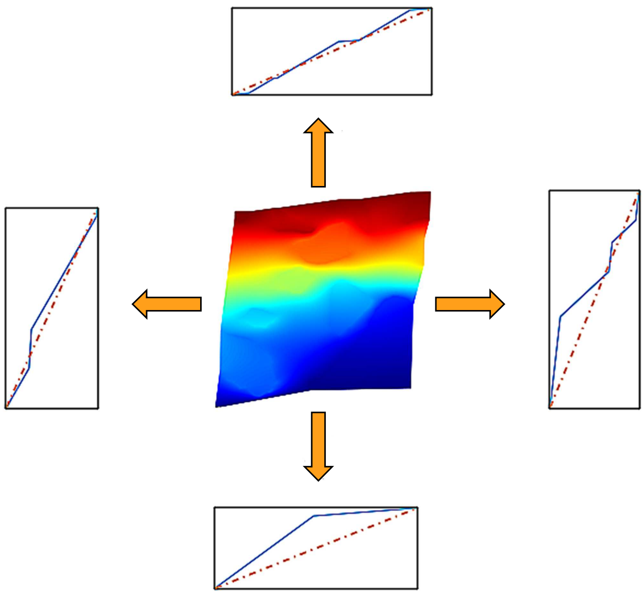

Fig. 4. Modified Dirichlet boundary conditions used for a specific RVE test

Modifications of the boundary conditions are performed in the spirit of the multiscale finite element method (Klimczak and Cecot, 2020b). Namely, we solve auxiliary boundary value problems along the edges of the RVE, where at least two components can be distinguished.

Practically, in the examples presented in this paper, we used 4 types of Dirichlet boundary conditions. They are schematically shown in Fig. 3 and represent tensile and shear tests in both directions. We used the prescribed displacement of 5% for all four RVE tests. In the case of shear tests, it was the maximum value (c.f. Fig. 4).

On the basis of our experience with the multiscale finite element method, MsFEM (Klimczak and Cecot, 2020a,b), we were aware of the influence of heterogeneities occurring along the domain boundary on the homogenized solution. In terms of the MsFEM, we modify the standard shape functions in order to account for the domain (and also its boundary) heterogeneity. That method results in the assessment of effective macroscale stiffness matrices that contain information on the homogenized material properties.

In terms of the RVE-based homogenization methods, the influence of the heterogeneous boundary can be also accounted for using a buffer (homogeneous) zone with an experimentally adjusted width. This approach shares the similar observation that was the basis of our research. Namely, the subdomain occupied by the RVE (when the whole domain is subject to the load) does not exhibit the response being just a copy of the macroscale boundary conditions. Even in the case of the tensile test, the response within the domain is not linear due to its heterogeneity. Thus, it should be accounted for in the RVE analysis. Motivated by the MsFEM approach, we modify Dirichlet boundary conditions used for all 4 RVE tests. Considering the enforced displacements (see Fig. 3) as functions ψ, we propose below the method for their modifications that covers the microstructure heterogeneity along every RVE edge. Graphically, the idea is shown in Fig. 4 on the example of a shear test. Therein, one can observe the effect of our algorithm performance with respect to the standard boundary conditions used for the corresponding RVE test.

Given ψ, which is a standard scalar function describing Dirichlet boundary conditions, we look for its scalar-valued counterpart φ that is a discrete solution of the following Dirichlet boundary value problem with $\varphi \in C^0(\Omega)$

$$\begin{aligned} &\frac{d}{ds}(2\mu+\lambda)\frac{d\varphi}{ds} = 0 \qquad \forall s \in (0,l) \\ &\varphi = \psi \qquad \text{on} \quad \partial\Omega \end{aligned} \tag{2.3}$$

where μ and λ are Lamé constants, Ω is the analyzed RVE domain and l is length of the analyzed RVE edge.

Schematically, the proposed framework consists of the following steps:

- Generation of a set of RVE's.
- Solution of the auxiliary boundary value problems along the RVE edges with two phases present.
- Solutions of the microscale numerical tests with modified boundary conditions for every RVE, which leads to assessment of the effective parameters tensors $\mathbf{C}_{eff}$.
- Solutions of the macroscale problem with effective parameter tensors used at the integration points.

For the numerical tests presented in this paper, we used also some additional assumptions. In order to provide a reliable comparison between direct and multiscale approaches, we proceed as follows:

- A coarse mesh is generated at the macroscale level.
- Each of the coarse mesh elements is treated as the RVE i.e., a mesh refinement is performed within every RVE in order to account for the underlying microstructure.
- Effective parameters tensors $\mathbf{C}_{eff}$ are assessed for every RVE/coarse mesh element.
- The macroscale problem is solved using these effective tensors for integration.

For the sake of further comparison, for direct analysis we globally generate a fine mesh that is a union of fine meshes generated within RVE's/coarse mesh elements. In such a way, the asphalt concrete microstructure is accounted for with the same precision for both the direct and multiscale analysis. At the macroscale level, we use the higher order approximation to increase the accuracy of the solution.

3. Numerical results

In order to illustrate the efficiency of the proposed approach, we present two numerical tests: with a periodic AC synthetic microstructure and with a non-periodic one recognized from a high-quality image. For the sake of simplicity, we use dimensionless quantities for the tests.

3.1. Test 1: periodic AC microstructure

The periodic microstructure generated for this test was prepared according to the procedure presented in (Klimczak and Cecot, 2020b). Namely, a gradation curve for the aggregate is selected first. Then, a packing of spheres/circles algorithm is used to populate the oversampled domain with spheres of diameters corresponding to the gradation curve. Subsequently, only the centers of spheres/circles belonging to the analyzed domain (a subdomain of the oversampled domain) are left. They are copied in a 3x3 pattern and serve as seeds of Voronoi tessellation. Finally, only the microstructure cut out from the analyzed domain is left (see Fig. 5). Such an RVE local microstructure can be copied in both directions creating a periodic microstructure.

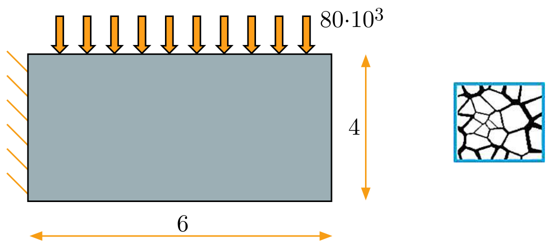

Fig. 5. Macroscale boundary value problem (left) and the periodic RVE (right)

In this test, we created an exemplary microstructure consisting of 24 RVE's shown in Fig. 5. Boundary conditions are also presented therein. The problem is analyzed on the assumption that both the aggregate particles and the binder are linear elastic. Additionally, the plane strain state is assumed. Young's moduli are equal to $80 \cdot 10^9$ (aggregate) and $10 \cdot 10^9$ (binder). Poisson's ratios are equal to 0.35 (aggregate) and 0.3 (binder). In Fig. 6, the domain microstructure obtained by copying the RVE in a 4×6 pattern and the corresponding fine mesh are shown. Since all RVE's exhibit the same microstructure, the tensor of effective parameters was assessed only once and used for every coarse mesh element.

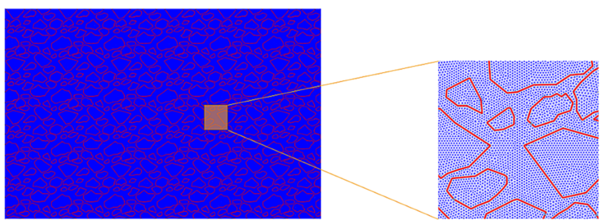

Fig. 6. Domain microstructure and the zoomed-in fine mesh

In Fig. 7, we present the comparison between the reference solution (top row, $p = 1$, NDOF $=$ 500000) and the homogenized one (central row) obtained for the approximation order $p = 1$ at the macroscale. Additionally, the absolute error is presented in the bottom row. The coarse mesh consists of 24 rectangular elements.

It should be emphasized that the reference solution was obtained using approximately half a million degrees of freedom. The error convergence for this test is shown in Fig. 8. It can be observed that a substantial reduction of degrees of freedom was obtained in the test even for the linear approximation used at the macroscale level. Modifications of the boundary conditions

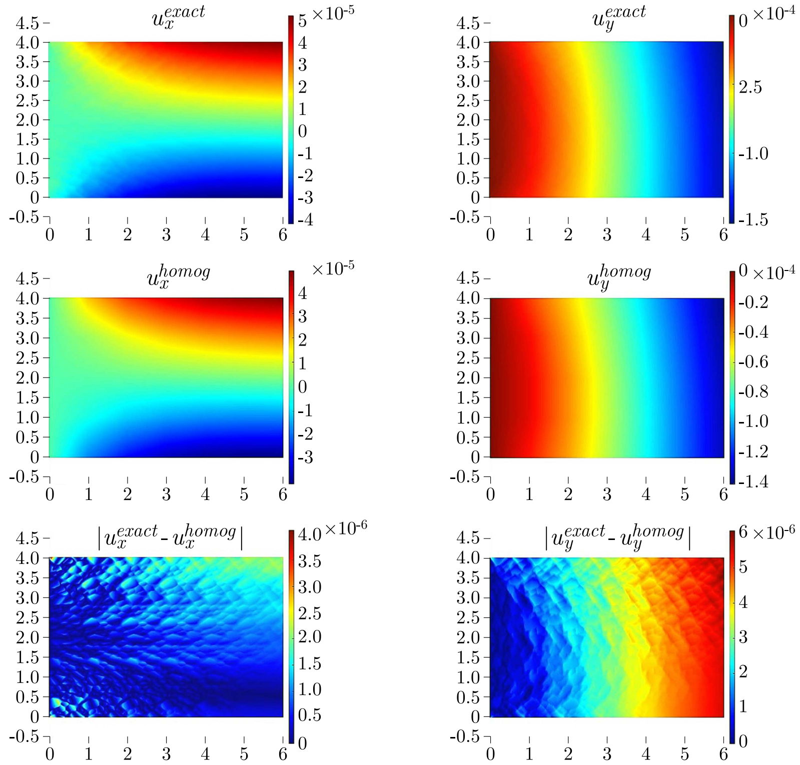

Fig. 7. Comparison between the reference and selected homogenized solutions – Test 1 (reference solution - top row, homogenized solution for the linear approximation at the macroscale – central row, absolute error – bottom row)

in the RVE tests increased the accuracy of the results. It can be seen that the convergence rate for the "modified BC's" solution (blue line) is faster than for the "standard BC's" solution (remaining lines).

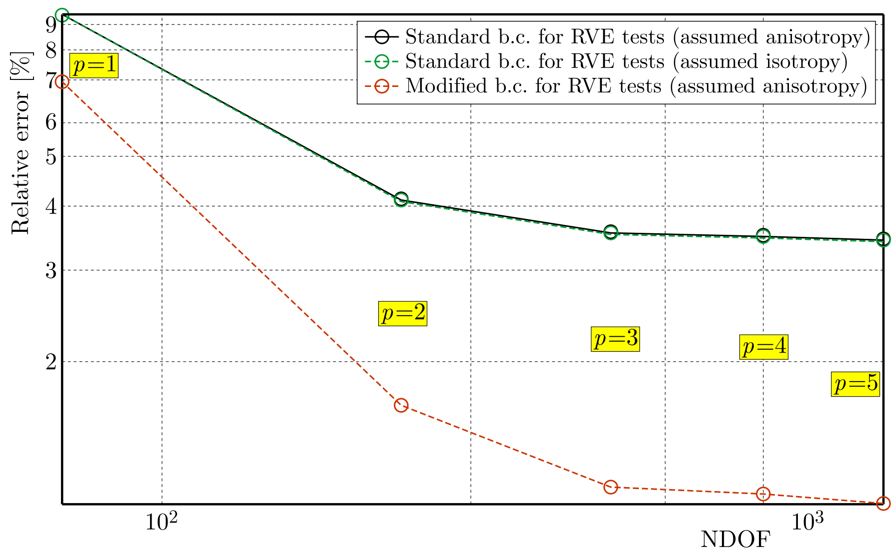

Fig. 8. Error p-convergence (logarithmic scale)

For the standard boundary conditions used in RVE tests, we additionally examined the influence of the assumption on anisotropy/isotropy of asphalt concrete. This effect was negligible (green and black lines).

3.2. Test 2: non-periodic AC microstructure

In this test, we generated the AC microstructure from a high-quality image of the real specimen. The scheme of image processing used for the microstructure recognition is presented in Fig. 9. The AC image shown therein is the actual one used in this test. It is also the case of the resulting microstructure geometry.

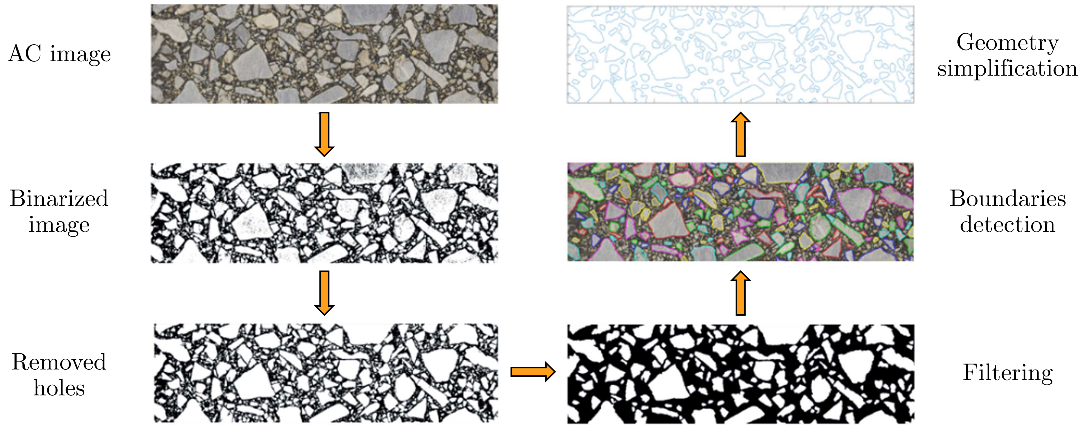

Fig. 9. Recognition of the AC microstructure using image processing

The first step in the image processing is image binarization. Small objects ("holes") within larger subdomains are removed subsequently. The next step is typically the filtering process that allows for further microstructure simplification. Namely, the objects with an area smaller than a threshold value are removed. Subsequently, the boundaries of aggregate particles are detected. At this level, the microstructure geometry can be further simplified due to possible boundaries processing. For the sake of this test, we used a moderate simplification in order to avoid excessively dense finite element mesh.

Material data for this test are the same as for the previous one, whereas the boundary conditions and domain dimensions are different (see Fig. 10).

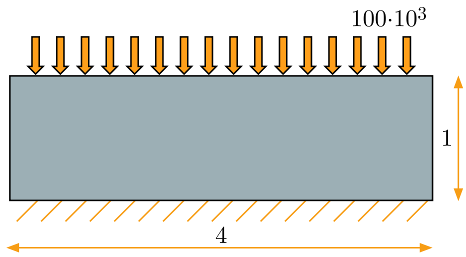

Fig. 10. Analyzed domain

Since the microstructure geometry is non-periodic, tensors of effective material parameters were assessed for each coarse mesh element independently. In this test, we also investigated the effect of the increasing approximation order at the macroscale level. In addition, we generated 2 coarse meshes consisting of 1×4 and 2×8 square elements, respectively. It is to present the necessity of careful RVE size selection.

In Fig. 11, the comparison between the reference and the homogenized solutions obtained for 2×8 coarse elements discretization at the macroscale is presented for the linear approximation used at both levels. In order to obtain the reference solution, the problem consisting of approximately 100000 degrees of freedom had to be solved.

Qualitatively, all of the homogenized solutions (these shown in Fig. 11 and those skipped for the sake of brevity) are of an acceptable form. Quantitative analysis can be performed using

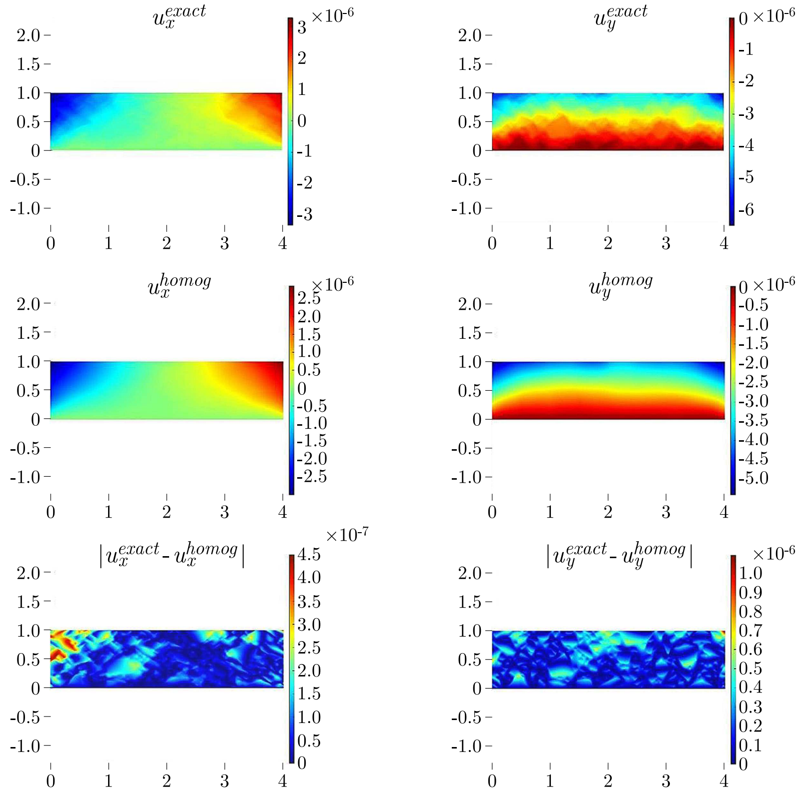

Fig. 11. Comparison between the reference and selected homogenized solutions – Test 2 (reference solution – top row, homogenized solution for the linear approximation at the macroscale – central row, absolute error – bottom row)

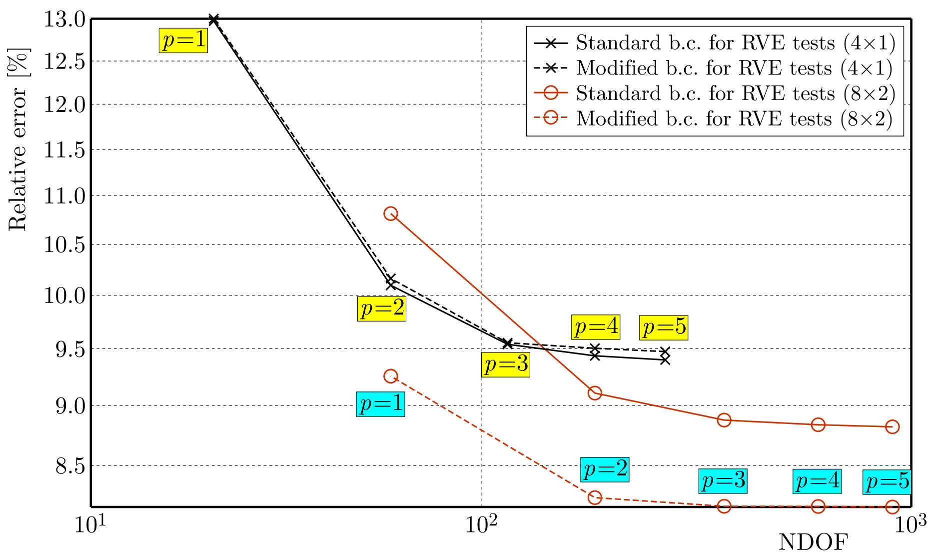

Fig. 12. Error p-convergence (logarithmic scale)

p-convergence for both coarse discretizations (shown in Fig. 12). We present in Fig. 12 the homogenized results obtained both for the standard and modified Dirichlet boundary conditions at the RVE level analysis. The difference between these curves for the coarser (1×4 elements) discretization is very small. The introduced modifications cause even a slight deterioration of the homogenized solution accuracy. The results for this discretization are shown in order to emphasize the necessity of careful selection of the RVE size. All of the approaches based on the

RVE concept require a distinct separation of scales. It refers to two cases: the ratio of the RVE size and the domain size as well as the ratio of the inclusion size to the RVE size. In practical applications, the coarser (1×4 elements) discretization would not be used due to violation of separation of the scales condition. For a more justified discretization (2×8 elements), the modification of Dirichlet boundary conditions at the RVE level improved the solution accuracy.

4. Discussion

Every newly developed numerical method should be effective in a sense of computational resources consumption. The numerical homogenization is evidently a reliable and efficient method. In the tests presented in the previous Section, it allowed for a substantial reduction of the number of degrees of freedom necessary in the analysis. Instead of a direct approach leading to a 500000-NDOF (Test 1) or 100000-NDOF (Test 2) problem, one can solve an equivalent 70-NDOF (Test 1, $p = 1$) or 20-NDOF (Test 2, $p = 1$, 1×4 finite elements) problem. Analyzing the introduced additional modeling error, approximately 7% and 13% for these two cases were measured in L_2 norm, respectively. In some initial tests, even such values are of an acceptable order. As it was demonstrated in Figs. 8 and 12 that the solution accuracy can be easily improved by a higher order approximation and modification of Dirichlet boundary conditions in the RVE analysis.

The numerical effectiveness of the proposed framework can be summarized as follows:

- RVE analyses are performed independently, hence this process can be parallelized.
- Assessment of the effective tensors of material parameters is performed only once, an increase of the approximation order at the macroscale does not require any updates, since standard shape functions are used.
- For a periodic microstructure, only one effective tensor of material parameters needs to be assessed.
- Modification of the boundary conditions for the RVE analysis is relatively cheap, since it is performed only along the boundaries.

5. Conclusions

In this paper, we presented the application of the newly developed version of numerical homogenization to linear elastic analysis of asphalt concrete. The main novelties of the proposed approach are as follows:

- Application of a higher order approximation at the macroscale level.
- Enhancement of the standard numerical homogenization due to modification of Dirichlet boundary conditions used for RVE analysis.
- Solution of two numerical tests with periodic and non-periodic asphalt concrete microstructures.

The obtained results confirmed the applicability of the developed method to linear elastic analyses of asphalt concrete. Precisely, we demonstrated that a reduction of the number of degrees of freedom by several orders of magnitude introduced the error at the acceptable level (maximum 13% was measured).

Our future research plan is to apply this approach to nonlinear analyses of asphalt concrete. In the limit of small displacements, we developed some alternative methods (Klimczak and Cecot, 2020a; Oleksy and Cecot, 2015). Therein, weak formulations of nonlinear problems were expressed in such a way that the terms corresponding to inelastic strains were added to the right--hand side (load vector). The stiffness matrix remained the same for all nonlinear iterations. In terms of numerical homogenization, it means that the most time-consuming computations of $\mathbf{C}_{\mathit{eff}}$ would be performed only once, as for the linear elasticity.

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